A VARIATION-GRADIENT METHOD FOR OPTIMIZATION OF THE SHAPE OF BLADE CASCADES

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A variation-gradient method for solving the problem on optimization of the shape of blade cascades has been developed and tested. This method is based on calculation of the gradient of the functional of the optimization problem with the use of the parameters of a fluid flow varied relative to the design parameters determined from the system of variation gasdynamic equations. The efficiency of the indicated method was tested, as compared to the efficiency of the analogous gradient method in which the gradient of the functional of the optimization problem is calculated using the finite-difference method, in solving concrete problems on optimization of blade cascades. It is shown that the method proposed allows one to substantially decrease the time of determining the extremum of the indicated functional.

Hydrodynamic cascades are used as models of turbo-machines (turbines, vane compressors, impeller pumps, fans) when the problem on upgrading of the quality of these machines is being solved. Since it is precisely the character of hydrodynamic interaction of the hydrodynamic cascade of a turbo-machine with a fluid flow (which is mainly dependent on the cascade geometry) that determines the quality of the turbo-machine, a pressing problem is the development and improvement of methods for optimization of the shape of blade cascades.

At present, in the process of designing the blade cascades of turbomachines, i.e., selecting the geometric parameters of the cascades providing definite characteristics of the turbomachines, the geometry of blade cascades is disturbed relative to the known geometry of a prototype with the use of its experimental and theoretical aerohydrodynamic characteristics. It is difficult to improve a blade cascade with the use of this method because, in this case, it is necessary to exhaust a large number of combinations of geometric parameters.

There are two other approaches for solving the indicated problem, which are more efficient. The first approach combines different methods of solving the inverse problems on the aerodynamics and optimization of the shape of streamlined bodies in the process of designing them. These methods, having a long history (see, e.g., [1–3] and the bibliography therein), are being developed further at present [4, 5]. An unquestionable merit of the indicated approach is its high efficiency, which is attained with the use of the developed analytical description of fluid flows; however, it is mainly applied to the plane case. The second approach combines different methods of numerical optimization of blade cascades on the basis of solution of direct problems on fluid flows around them with the use of iteration algorithms for solving hydrogasdynamics problems [6–9]. The advantage of this approach is the possibility of using more complex models of flows because modern programs for numerical simulation of three-dimensional flows in industrial plants allow one to calculate the characteristics of a definite-geometry flow in the viscous-fluid approximation with account for the turbulence effects. However, these calculations are time-consuming, which prevents the use of such algorithms for solving optimization problems. To obviate the indicated difficulties, it is necessary, on the one hand, to devise new algorithms for calculating fluid flows and modify the existing ones and, on the other, to develop and modify algorithms for solving optimization aerohydrodynamics problems. This is the challenge of the present work.

The most general formulation of the problem on automatic optimization of the shape of units of a turbo-machine is as follows:

1) parametric description of the geometry of a blade cascade, which should be correct from the hydrodynamics standpoint in the case where parameters are varied and plausible from the engineering standpoint;

2) determination of the purposeful functional and the functional restrictions for the optimization problem;

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3) calculation of the fluid flow in the flow passage of the turbo-machine, including the fluid flow in the rotating impeller;

4) solution of the extremum problem with selection of an efficient strategy of search for an optimum solution.

Since the prime objective of the present work is comparative analysis of the efficiency of different strategies for solving problems on optimization of the aerohydrodynamics of blade cascades, it makes sense to consider a maximum simple problem that, however, will retain the specificity warranting the use of the method developed.

The following simplifications were used in the formulation of the problem:

1. It is assumed that the cross sections of the blades considered are plane, which makes it possible to determine the outline of a blade in the standard parametric way with the use of the mean line and the thickness function. We have three varied parameters — the sag of the mean line f, its abscissa x_f, and the maximum thickness of the blade c. The length of the chord of the blade b and the abscissa of its maximum thickness x_c are assumed to be constant. The thickness function was selected from the series of NACA blades:

$$L(x) = c\left(0.2969x^{1/2} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4\right),$$

the mean line is parabolic in shape.

2. The viscosity of a fluid, the drag coefficient of the blade, and the nonseparation criterion are determined using the integral relations for a two-dimensional boundary layer, which were described and substantiated, e.g., in [10, 11]. The ratio between the lift coefficient and the drag coefficient of the blade, i.e., its lift–drag quality, is used as a functional. The functional restrictions require that the lift coefficient be of definite value and the nonseparation criterion be fulfilled.

3. In accordance with items 1 and 2, the flow in a blade cascade is calculated on the basis of the gasdynamics equations for a two-dimensional fluid flow:

$$\mathbf{U}_t + \mathbf{F}_x + \mathbf{G}_y = 0\,,$$

$$\mathbf{U} = (\rho, \rho u, \rho y, e)\,, \quad \mathbf{F} = (\rho u, p + \rho u^2, \rho u y, u(p + e))\,, \quad \mathbf{G} = (\rho v, \rho u v, p + \rho v^2, v(p + e))\,. \tag{1}$$

The formulation of the boundary problem for system (1) is closed by setting of the impenetrability condition on the surface of the blade cascade

$$L(x, \beta) = 0\,, \tag{2}$$

and the determination of the velocity vector at the left boundary of the computational region and the pressure at the right boundary of the computational region, and setting of the periodicity conditions at the upper and lower boundaries outside the blade outline.

4. The extremum problem is solved by the steepest-descent method [12] and the variation-gradient method formulated in [13] and refined in the present paper. The last-mentioned method is undoubtedly also among the steepest-descent methods; it differs from their traditional variant used for solving optimization problems of aerohydrodynamics by the relations used for calculating the gradient of the functional of the problem.

The gradient of the functional J of the optimization problem has the form

$$\nabla_\beta J = J_{\mathbf{P}} \mathbf{P}_\beta\,,$$

where $\mathbf{P} = (u, v, \rho, p)$ and the variations in the parameters of a fluid flow are determined in accordance with the geometric parameters from the solution of the system of linear differential variation equations:

$$(\rho_\beta)_t + \operatorname{div}(\rho\mathbf{u})_\beta = 0\,,$$

$$((\rho\mathbf{u})_\beta)_t + ((\rho\mathbf{u})_\beta \cdot \nabla)\,\mathbf{u} + ((\rho\mathbf{u}) \cdot \nabla)\,\mathbf{u}_\beta + \nabla(p_\beta) = 0\,, \quad \operatorname{div}[(e + p)\,\mathbf{u}]_\beta = 0\,. \tag{3}$$

The boundary conditions for system (3) for the blade outline are the relations

TABLE 1. Values of the Functional J for the Optimization Problem

Problem	T	Steepest-descent method	Variation-gradient method
No. 1	T_{50}	34.4	39.9
No. 1	T_{100}	41.9	45.2
No. 2	T_{50}	20.3	53.7
No. 2	T_{100}	63.0	69.5

TABLE 2. Initial and Optimum Values of Varied Parameters and Corresponding Values of the Lift Coefficient and Functional

Problem	T	α	x_f	f	c_x	c_y
Nos. 1, 2	T_0	0.00	0.500	0.000	0.600	0.000
No. 1	T_{50}	1.21	0.483	0.138	0.464	0.491
No. 1	T_{100}	2.48	0.479	0.126	0.455	0.498
No. 2	T_{50}	14.8	0.532	0.187	0.600	0.802
No. 2	T_{100}	15.2	0.486	0.174	0.420	0.797

$$\nabla L_\beta \cdot \mathbf{u} + \nabla L \cdot \mathbf{u}_\beta = 0 \,, \quad \beta = (f, x_f, c) \tag{4}$$

and the variations of the boundary conditions of system (1) at the other boundaries.

In the process of minimization of the functional for the purpose of determining its gradient, the boundary problem (3)–(4) is solved in each step at constant gasdynamic characteristics of a flow, determined from the solution of problem (1)–(2). The method proposed is more efficient because the time of solution of problem (3)–(4) is always smaller than the time of solution of problem (1)–(2). The ratio between the time of calculating the gradient by the method of one-sided finite differences and the analogous time in the method proposed is equal to $(1 + \lambda N_\beta)/(1 + N_\beta)$, and, for the central differences increasing the accuracy of calculating the gradient, this time is equal to $(1 + \lambda N_\beta)/(1 + 2N_\beta)$, where N_β is the number of varied parameters and $\lambda = T_{34}/T_{12}$ is the ratio between the times of solving problems (3)–(4) and (1)–(2) respectively. At $N_\beta \to \infty$, the limiting values to which the rate of calculation of the gradient functional can be increased are equal to $1/\lambda$ and $2/\lambda$ for these cases. The more complex the physical model of a fluid flow and the corresponding system of differential equations, the smaller the value of λ and the more efficient the algorithm.

The problem being considered is numerically solved with the use of finite-volume schemes and a 272 × 40 computational grid that is adapted geometrically to the outline of the blades of a cascade in the physical region and is bunched in the neighborhood of the blades; in the canonical region, this grid is rectangular. The integration with respect to time was carried out by the third-order Runge–Kutta method. This algorithm was tested in detail in [14] in solving problems on subsonic and transonic flows around airfoil sections.

The optimization problem was formulated in the following way: to construct the shape of a blade cascade, satisfying the following aerogasdynamic and geometric restrictions

$$q_0 = \text{const} \,, \quad c_y = c_y^0 \,, \quad F(s) \geq F_0 \,,$$

$$c_{\min} \leq c \,, \quad c \leq c_{\max} \,, \quad b = \text{const} \,,$$

for which the goal functional $J(\alpha, \boldsymbol{\beta}, \mathbf{g}) = c_y/c_x$, $\mathbf{g} = (q_0, c_y^0, F_0, c_{\min}, c_{\max})$ reaches a maximum.

Results of Calculations. Two methods were used for solving the following optimization problems:

$$1. \; J(\alpha, \boldsymbol{\beta}, \mathbf{g}) \Rightarrow \max \quad \text{at} \quad \mathbf{g} = (0.3, 0.5, -3, 4\%, 16\%) \,,$$

$$2. \; J(\alpha, \boldsymbol{\beta}, \mathbf{g}) \Rightarrow \max \quad \text{at} \quad \mathbf{g} = (0.3, 0.8, -3, 4\%, 16\%) \,.$$

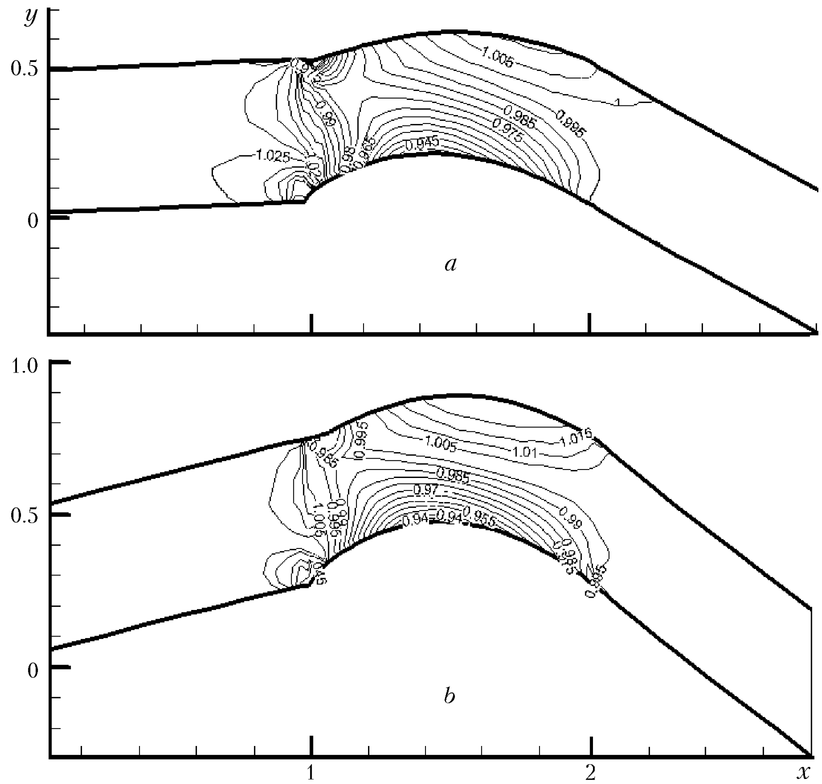

Fig. 1. Pressure field in the interblade region for the profile of a blade cascade, obtained as a result of solution of problem Nos. 1 (a) and 2 (b) at N = 100.

The varied parameters are α, f, x_f, and c.

The efficiencies of the methods used were compared for one and the same time of solving the optimization problem T_N corresponding to the time of solving problem (1)–(2) N times. In the calculations, it was assumed that N = 50 and 100. The results of solution of the problems formulated are presented in Table 1. In all the variants considered, the variation-gradient method allows one to obtain a deeper extremum. The initial and optimum values of the variables used in the variation-gradient method for different T_N are presented in Table 2. It is seen that there is a characteristic (for determination of the optimum shape of a blade) "play" between the angle of attack and the curvature for decreasing the value of c_x at a given restriction on c_y, for example, in the time interval from T_{50} to T_{100}, α increases and f decreases.

Figure 1 presents the isolines of the pressures in the computational region, and Fig. 2 shows the shape of a blade and the distribution of the pressure coefficient over the upper and lower surfaces of the optimum blade cascades for problem Nos. 1 and 2 at $T = T_{100}$, obtained by the variation-gradient method.

It should be also noted that, for the purpose of decreasing the time of solving the optimization problem in both methods, in solving the equations we used, as the initial approximation for the second and all subsequent calculations, the steady-state fields obtained in the previous optimization step. This allowed us to economize 30% of the calculation time.

Thus, the method proposed for solving optimization problems on the basis of calculation of the gradient of their functional with the use of the parameters of a fluid flow varied relative to the design variables determined from the system of gas-dynamics variation equations allows one to determine the extremum of this functional for a much shorter time. This effect will be enhanced with increase in the number of varied parameters.

It should be noted that, since the large dependence of the result on the choice of the initial point and the probability of reduction of the solution to a local extremum, characteristic of gradient methods, are retained in this case, nondeterminate methods are more preferential for problems with a large number of variables because they allow one to attain a global extremum. In this case, there is reason to use the approach proposed for rapidly obtaining a pre-

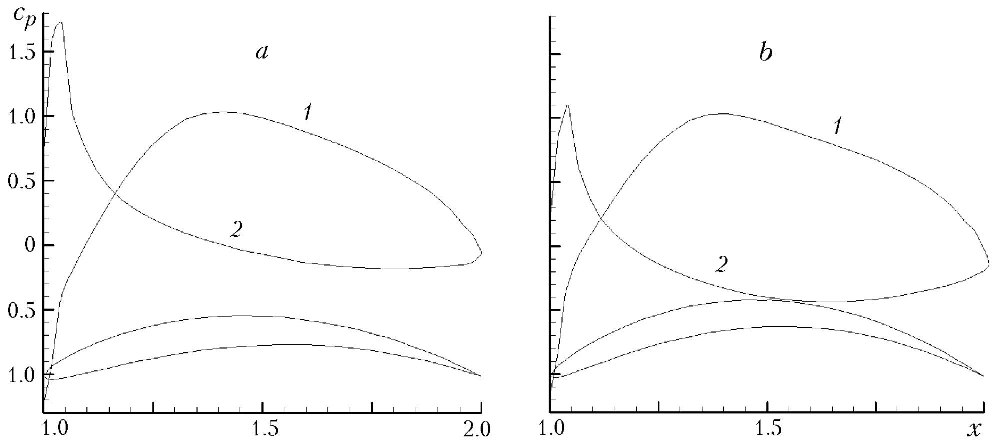

Fig. 2. Distribution of the pressure coefficient over the upper (1) and lower (2) outlines of the profile of the blade cascade, obtained as a result of solution of problem Nos. 1 (a) and 2 (b) at N = 100.

liminary solution and refine it then because the use of nongradient methods of determining the extremum of the functional from an arbitrary initial point requires, as a rule, a large number of calculations that are very time consuming in the case where the equations of motion are solved by iteration methods.

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NOTATION

a_0, dimensional velocity of sound in the incident flow, m/sec; a, velocity of sound relative to a_0; b, chord of a blade; $c_{\min}$ and $c_{\max}$, lower and upper boundaries of the maximum thickness of the blade, %; c_x, drag coefficient; c_y, lift coefficient; c_p, pressure coefficient; J, functional of the optimization problem; $F(s)$, form factor; F_0, constant involved in the nonseparation criterion selected; $\mathbf{g}$, vector of restrictions of the optimization problem; q_0, modulus of the incident flow velocity (Mach number of the incident flow in normalized variables); e, total energy of a unit gas volume normalized to the value of $\rho_0 a_0^2$; p, pressure normalized to the value of $\rho_0 a_0^2$; s, length of the arc along the blade outline; t, time relative to b/a_0; T_N, time of solution of the boundary problem N times; u, v, gas-velocity components relative to a_0; x and y, coordinates directed lengthwise and crosswise of the blade chord related to its length b; $\mathbf{F}$, $\mathbf{G}$, $\mathbf{U}$, vectors of the gasdynamic parameters of the flow; α, angle of attack; $\boldsymbol{\beta}$, vector of the geometric parameters; ρ_0, density of the incident flow; ρ, density related to ρ_0. Subscripts: 0, parameter of the incident flow; min, minimum; max, maximum.

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